

Lecture 13

Poisson's Distribution

Let X be a discrete random variable. X is said to follow Poisson's distribution with parameter λ if

$$P(X = r) = \frac{\lambda^r e^{-\lambda}}{r!}, \quad r = 0, 1, 2, \dots$$

$$\sum_{r=0}^{\infty} \frac{\lambda^r e^{-\lambda}}{r!} = 1$$

If the underlying situation is Binomial and X has no upper bound.

In Practice

- X : no. of occurrences of an event in unit time interval.
- λ : average no. of occurrences in unit time interval (in a hour).

Example

Whether a car passes or not, Bernoulli distribution because

$$1 \rightarrow \text{car } (X = 1), \quad 0 \rightarrow \text{no car } (X = 0)$$

Average no. of accidents on a road in a week = 3, i.e. $E(X) = 3$.

Example 1

Let X : no. of accidents in one week.

It is again Bernoulli as:

$$\text{Accident } (X) \rightarrow 1, \quad \text{No accident } (X) \rightarrow 0$$

$$P(X = 5)$$

For 5 accidents in one week:

$$P(X = 5) = \frac{3^5 e^{-3}}{5!}$$

Example 2

Let X : no. of patients having cancer.

$$\text{Yes} \rightarrow 1, \quad \text{No} \rightarrow 0$$

Since no upper limit, $\Rightarrow$ Poisson distribution.

Question 2

Probability of 5 accidents in 2 weeks:

$$\lambda = 6$$

$$P(Y = 5) = \frac{6^5 e^{-6}}{5!}$$

Note

If n is large and p is small, then the results generated from Binomial and Poisson are almost same.

Recurrence Formula

$$P(X = r) = \frac{\lambda^r e^{-\lambda}}{r!}$$

$$P(X = r + 1) = \frac{\lambda}{r + 1} P(X = r)$$

Expectation and MGF

$$E(X) = \sum_{r=0}^{\infty} r \cdot \frac{\lambda^r e^{-\lambda}}{r!} = \lambda$$

$$Var(X) = \lambda$$

Moment Generating Function

$$M_X(t) = \sum_{r=0}^{\infty} e^{rt} \cdot \frac{\lambda^r e^{-\lambda}}{r!}$$

$$= e^{-\lambda} \cdot e^{\lambda e^t} = e^{\lambda(e^t-1)}$$

$$M_X(t) = e^{\lambda(e^t-1)}$$