Lecture 3

2.15 Events and Joint Occurrence

Let $A \subset S$ (sample space). The joint occurrence of events A and B is given by $A \cap B$.

$$\text{If independent: } P(A \cap B) = P(A)P(B)$$
$$\text{If dependent: } P(A \cap B) = P(A)P(B|A)$$

2.16 Conditional Probability

If B has occurred first and then A occurs:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

where:

$$P(A) = \text{Probability of } A, \quad P(A|B) = \text{Probability of } A \text{ given } B.$$

Example 7. *A new student in IITJ is finding a class. The probability:*

$$P(\textit{Finding class} \mid \textit{Right-hand side}) = \dots$$

Given more and more information, uncertainty reduces. The process of updating information is called ***conditioning****.*

2.17 Multiplication Rule

$$P(A \cap B) = P(B)P(A|B)$$

As A occurs first, the **updation rule** increases in relevance.

2.18 Axioms for Conditional Probability

Check if $P(A|B)$ satisfies probability axioms:

1. $P(S|B) = 1$
2. If A_1 and A_2 are mutually exclusive then $P(A_1 \cup A_2 \mid B) = P(A_1 \mid B) + P(A_2 \mid B)$

Thus, conditional probability satisfies all the axioms of probability.

2.19 Law of Total Probability

If $B_1, B_2, \ldots, B_n$ is a partition of S, then for any $A \subset S$:

$$P(A) = \sum_i P(A|B_i)P(B_i)$$

The partition of S mean $S = \cup B_i$ and $B_i \cap B_j = \phi$, for all $i \neq j$.

Example 8. *Searching for a student in IITJ: partition IITJ into subsets like hostel, mess, etc., and search in each section separately.*

Example 9 (Factory with 3 Units)**.** *Three production units:*

- *Unit 1:* 50% *capacity,* 5% *defective*
- *Unit 2:* 30% *capacity,* 3% *defective*
- *Unit 3:* 20% *capacity,* 1% *defective*

1	***2***	***3***
50%, 5%	*30%, 3%*	*20%, 1%*

The probability that a product is defective:

$$P(D) = P(D|1)P(1) + P(D|2)P(2) + P(D|3)P(3)$$

Example 10 (Binary Communication Channel)**.** *The channel receives 0 and 1 with probabilities* 60% *and* 40% *respectively. Correct transmission rates:* 95% *for 0,* 90% *for 1.*

Transmitted	*Received*	*Probability*
0	*0*	*0.95*
0	*1*	*0.05*
1	*0*	*0.10*
1	*1*	*0.90*

We can compute:

$$P(\textit{0 received}) = P(\textit{0 received} \mid 0 \textit{ transmitted})P(0) + P(\textit{0 received} \mid 1 \textit{ transmitted})P(1)$$

This approach can be extended to n sequential occurrences.

2.20 Bayes' Theorem

If an item is found defective, we can ask: "Where did it come from?" — the place of maximum likely.

$$P(I|D) = \frac{P(I \cap D)}{P(D)} = \frac{P(D|I)P(I)}{\sum_i P(D|I_i)P(I_i)}$$

Theorem 2 (Bayes' Theorem)**.**

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$$

which implies:

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$