Indian Institute of Technology Jodhpur

Probability, Statistics and Stochastic Processes

MAL2010 (Practice Assignment 1)

1. Let $S = [0; 1]$, adding as few sets as possible, complete the family of sets $\{\phi, [0, 1/4), \{1\}\}$ to obtain a sigma algebra.

2. Let A and B be two events with probabilities $P(A) = 1/2$ and $P(B^c) = 1/4$. Can A and B be mutually exclusive events?

3. In the discrete space $S = \{1, 2, 3, 4, 5, 6\}$, suppose each point has probability 1/6. If $A_1 = \{1, 2, 3, 4\}$ and $A_2 = A_3 = \{4, 5, 6\}$ then $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$ but the events are not independent.

4. Let $S = \{1, 2, 3, 4, 5, 6\}$ with uniform probability. Show that if $A, B \in S$ are independent and A has 4 elements, then B must have 0, 3 or 6 elements.

5. A student of IIT goes to one of the two chai shops on campus, choosing Pan-Chai-Tea 60% of the time and Jeetu's shop 40% of the time. Regardless of where he goes, he buys a tea 70% of his visits.

   (a) The next time he goes into a chai shop, what is the probability that he goes to Pan-Chai-Tea and orders a tea?

   (b) Are the two events, going to Pan-Chai-Tea and ordering a tea are independent?

   (c) If he goes into a chai shop and orders a tea, what is the probability that he is at Jeetu?

   (d) What is the probability that he goes to Pan-Chai-Tea or orders a tea or both?

6. A survey of people in a given region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was roughly 10 times the probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the region is .006, what is the probability of death due to lung cancer given that a personn is smoker?

7. Suppose that, in a particular city, airport A handles 50% of all airline traffic, and airports B and C handle 30% and 20%, respectively. The detection rates for weapons at the three airports are 0.9, 0.8 and 0.85 respectively. If a passenger at one of the airports is found to be carrying a weapon through the boarding gate, what is the probability that the passenger is using airport A? AirportC?

8. Two players A and B draw balls one at a time alternatively from a box containing m white balls and n black balls. Suppose the player who picks the first white ball wins the game. What is the probability that the player who starts the game will win?

9. A simple binary communication channel carries messages by using only two signals, say 0 and 1. We assume that, for a given binary channel, 40% of the time a 1 is transmitted; the probability that a transmitted 0 is correctly received is 0.90, and the probability that a transmitted 1 is correctly received is 0.95. Determine (a) the probability of a 1 being received, and (b) given a 1 is received, the probability that 1 was transmitted.

10. Five percent of patients suffering from a certain disease are selected to undergo a new treatment that is believed to increase the recovery rate from 30 percent to 50 percent. A person is randomly chosen from these patients after the completion of treatment and is found to have recovered. What is the probability that the patient recceived the new treatment?

11. Four teams A, B, C and D compete in a tournament, and exactly one of them will win the tournament. Teams A and B have the same chance of winning the tournament. Team C is twice as likely to win the tournament as team D. The probability that either team A or team C wins the tournament is 0.6. Find the probability of team D winning the tournament.

12. Let X be a random variable with values $\{0, \pm 1, \pm 2\}$. Suppose that $P(X = -2) = P(X = -1)$ and $P(X = 1) = P(X = 2)$ with the information that $P(X > 0) = P(X < 0) = P(X = 0)$. Find the probability mass function and distribution function of the random variable.

13. Let X be a random variable with probability mass function $P(X = x) = (e^{-2}2^x)/x!, x = 0, 1, 2, \cdots$. Find the values of $E(X), E(X^2), E(X^3)$ and $E(X^4)$.

14. Suppose the random variable X has probability function $p(x), x = 0, \pm 1, \pm 2, \cdots$, find the probability mass functions of the transformations of $X, X^2, |X|$. Prove that $E(X - a)^2$ is minimized with respect to a when a= E(X).

15. Suppose you choose a real number X from the interval $[2, 10]$ with a density function of the form

$$f(x) = cx$$

where c is a constant.

(a) Find the value of c

(b) Find $P(X > 5), P(X < 7)$ and $P(X^2 - 12X + 35 > 0)$

(c) Find the mean and variance of X and $Y = X/2 - 1$.