

Indian Institute of Technology Jodhpur

Probability, Statistics and Stochastic Processes

MAL2010 (Practice Assignment 2)

1. Two people arrive to meet in the park. Each person is equally likely to arrive, independent of the other, at 2:00, 2:30 and 3:00 PM. Let X equal the time that the first person to arrive has to wait, where X is taken equal to 0 if both people arrive at the same time.

(a) What are the possible values of X ?

(b) What are the probabilities that X assumes each of these values?

2. A coin is biased in such a way that $P(H) = 3/7$ and $P(T) = 4/7$. Suppose that the coin is tossed three consecutive times and let X be the random variable that indicates the number of heads obtained. Find the distribution function of the random variable X and calculate $E(X)$.
3. A continuous random variable X has density function given by

$$f(x) = kxe^{-\lambda x}, x \geq 0$$

Determine the value of k . Also, find the distribution function of X .

4. The random variable X has probability density function

$$f(x) = \frac{8}{\pi} \sqrt{x(1-x)}, \quad 0 < x < 1$$

Obtain the density of $Y = 2X - 1$.

5. Let X be uniformly distributed over $(0, 4)$, that is, the pdf of X is $f(x) = 1/4$, $0 < x < 4$. Find the pdf of $Y = (X - 3)^2$.
6. Let X be a random variable with distribution

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, -\infty < x < \infty$$

Define $Y = e^X$. Find the pdf of Y .

7. Let X be a uniformly distributed random variable on the interval $(0, 4)$. Calculate the probability that the roots of

$$2y^2 + 2yX + (X + 1) = 0$$

are both complex.

8. Let X be a random variable with its density function

$$f_X(x) = 12x^3 - 21x^2 + 10x, \quad 0 \leq x \leq 1.$$

Calculate the expected value and the variance of X .

9. One thousand tickets are sold at Rs 1 each for some charity. Tickets are to be drawn at random and cash prizes to be awarded as; 1 prize for 100 Rs; 3 prizes for 50 Rs and 5 prizes for 20 Rs. What is your expected gain when you buy a ticket?
10. A dart is thrown randomly inside a circle of radius 5. You can get a prize worth Rs 100000 by hitting the centre of the circle. If the dart hits inside the concentric circle of radius 1, you receive Rs 10,000. If it hits the ring between circles of radius 1 and 2, you receive Rs 5,000. If it hits the ring between circles of radius 2 and 3, you receive Rs 2,000. If it hits in any other part of the circle you have to pay some amount a . How much amount a would you be paying for this game to become fair?
11. Let θ be a randomly chosen angle in $(0, \pi/4)$. The random variable Y is defined as the y-coordinate of the point at which the ray through the origin at angle θ intersects the line $x = 1$ in the plane. What are the expected value and the standard deviation of the area of the triangle with the corner points $(0, 0)$, $(1, 0)$ and $(1, Y)$?
12. Let X be a random variable with density function given by: $f(x) = x^3, 0 < x \leq 1$ and $f(x) = (2 - x)^3, 1 < x \leq 2$
- (a) Calculate $\mu = E(X)$ and $\sigma^2 = Var(X)$.
- (b) Find $P(\mu - 2\sigma < X < \mu + 2\sigma)$.