

Indian Institute of Technology Jodhpur

Probability, Statistics and Stochastic Processes

MAL2010 (Practice Assignment 3)

1. Suppose that X is a random variable having geometric distribution with parameter p . Find the probability mass function of X^2 and $X + 3$.
2. Let X be a random variable with mean 11 and variance 9. Use Chebychev's inequality to find (i) a lower bound for $P(6 < X < 16)$ (ii) The value of k such that $P(|X - 11| \geq k) \leq 0.09$.
3. Let X be a random variable with characteristic function given by

$$\phi_X(t) = 1/7(2 + e^{(-it)} + e^{it} + 3e^{2it})$$

Determine $P(-1 \leq X \leq 1/2)$ and $E(X)$.

4. Let X be a random variable with characteristic function given by $\phi_X(t) = e^{(2(e^{it}-1))}$. Determine $E(2X^2 - 5X + 1)$.
5. Let X be a random variable having binomial distribution with parameters n and p . Prove that

$$E(1/(X + 1)) = (1 - (1 - p)^{(n+1)})/(n + 1)p.$$

6. Let X be a random variable having a Poisson distribution with parameter λ . Prove that

$$E(X^n) = \lambda E((X + 1)^{(n-1)}).$$

7. Let R be the rate at which customers are served in a queue. Suppose that R has density $f(r) = 2e^{-2r}, r > 0$. Find the pdf of the waiting time per customer $T = 1/R$.
8. A communication system consists of n components, each of which independently functions with probability p . The total system will be able to operate effectively if at least one-half of its components function. For what probability p is a 5-component system more likely to operate effectively than a 3-component system?
9. Obtain MGF for the Laplace pdf $f(x) = \frac{1}{2\lambda} e^{-|x-\mu|/\lambda}; x \in R$

10. A target is made of three concentric circles of radii $1/\sqrt{3}$, 1 and $\sqrt{3}$ meters. Shots within the inner circle count 4 points, in the next ring 3 points and within the third ring 2 points (Shots outside the target count zero). The distance of a shot from the center of the target is a random variable R with its density

$$f(r) = \frac{2}{\pi(1+r^2)}, \quad r > 0$$

Calculate the expected value of the score after five shots.