

# Indian Institute of Technology Jodhpur

## Probability, Statistics and Stochastic Processes

### MAL2010 (Practice Assignment 4)

1. Find (i)  $P(X > 5)$ , (ii)  $P(2 < X \leq 6)$  and (iii)  $P(X > 5|X < 8)$  for each of the following cases -
  - (a)  $X$  follows Binomial distribution with  $n = 10$  and  $p = 0.35$ .
  - (b)  $X$  follows Poisson's distribution with  $\lambda = 5$ .
  - (c)  $X$  follows Geometric distribution with  $p = 0.25$ .
2. Let  $X$  be a continuous random variable with pdf  $f(x) = 4x^3, 0 < x < 1$ . Find  $P(X \leq 2/3|X > 1/3)$ .
3. Suppose that  $Y = -2X + 3$ . If we know  $E(Y) = 1$  and  $E(Y^2) = 9$ , find  $E(X)$  and  $Var(X)$ .
4. Suppose that  $X$  follows Geometric distribution with parameter  $p$ . Find  $E(1/2^X)$ .
5. Let  $X$  be a Poisson random variable with parameter  $\lambda$ . Assume  $P(X = 2) = 2P(X = 1)$ . Find  $P(X = 0)$ .
6. Let  $X$  be a discrete random variable with support  $\{0, 1, 2, \dots\}$ . Show that

$$E(X) = \sum_{k=0}^{\infty} P(X > k).$$

7. 50 faculty members live in Type B of IIT Jodhpur. The parking lot has the capacity for 25 cars. If each faculty member has a car with probability  $1/2$  (independently from other students), what is the probability that there won't be enough parking spaces for all the cars?
8. The number of emails that I get in a weekday (Monday through Friday) can be modeled by a Poisson distribution with an average of  $1/6$  emails per minute. The number of emails that I receive on weekends (Saturday and Sunday) can be modeled by a Poisson distribution with an average of  $1/30$  emails per minute.
  - (a) What is the probability that I get no emails in an interval of length 4 hours on a Sunday?

- (b) A random day is chosen (all days of the week are equally likely to be selected), and a random interval of length one hour is selected on the chosen day. It is observed that I did not receive any emails in that interval. What is the probability that the chosen day is a weekday?
9. Suppose that  $X$  follows uniform distribution in  $[a, b]$ , and that  $E(X) = 2$  and  $Var(X) = 3/4$ . Calculate  $P(X \leq 1)$ .